. . *

Sombor Gutman

[12] [13]

Sombor

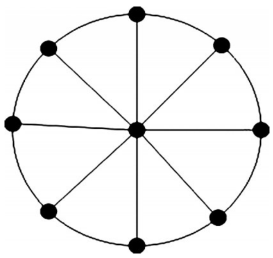
Sombor

1 [14] $(\square) = () \times ()$ $(_1, _1)$

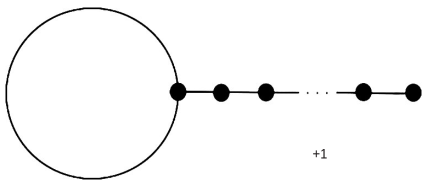
$(v_2, _2)$ $_1 = _2$ $_1 v_2 2 _2$ $($ $_1 = _2$ $_1 _2$ $()$ 1

2 [15] $(\times) = () \times ()$

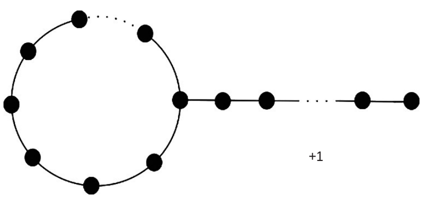
7 , + 1 + 1 6
8 + 1 + 1(≥ 1)
+ 1 , , 7



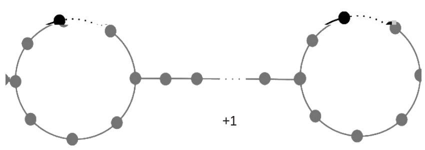
4 9



5 ,



6 ,



7 , ,

1 Sombor

Sombor

1 = □ Sombor

$$(\quad) = (11 \quad - 9 \quad - 20 \quad + 14)\sqrt{2} + 2\sqrt{2 \quad ^2 + 2 \quad + 1}$$

$$+(\quad - 2)(\quad - 1)\sqrt{\quad ^2 + 2 \quad + 26} + 2(\quad - 1)\sqrt{41} + 2(\quad - 1)\sqrt{\quad ^2 + 16}$$

$''(\quad) = \quad (\quad) = 3 \quad - \quad - 2$. $= 4$ $= \quad + 1$ 2

$(\quad - 2)(\quad - 1)$

$= 2\sqrt{(\ + 1)^2 + \ ^2} + (\ - 2)(\ - 1)\sqrt{(\ + 1)^2 + 5^2} + (2 \ - 5)(\ - 1)\sqrt{5^2 + 5^2}$

$+2(\ - 1)\sqrt{4^2 + 5^2} + 2(\ - 1)\sqrt{4^2 + 4^2} + 2(\ - 1)\sqrt{4^2 + \ ^2} + (\ - 3)\sqrt{(\ + 1)^2 + (\ + 1)^2}$

$= 2\sqrt{2 \ ^2 + 2 \ + 1} + (\ - 2)(\ - 1)\sqrt{ \ ^2 + 2 \ + 26} + 5(2 \ - 5)(\ - 1)\sqrt{2} + 2(\ - 1)\sqrt{41}$

$+8(\ - 1)\sqrt{2} + 2(\ - 1)\sqrt{16 + \ ^2} + (\ - 3)(\ + 1)\sqrt{2}$

$= (11 \ - 9 \ - 20 \ + 14)\sqrt{2} + 2\sqrt{2 \ ^2 + 2 \ + 1} + (\ - 2)(\ - 1)\sqrt{ \ ^2 + 2 \ + 26}$

$+2(\ - 1)\sqrt{41} + 2(\ - 1)\sqrt{ \ ^2 + 16}$

2 = □ Sombor

$(\ ) = (11 \ - 26 \ - 20 \ + 26)\sqrt{2} + 2\sqrt{2 \ ^2 + 2 \ + 1} + 2(\ - 2)\sqrt{ \ ^2 + 2 \ + 17} + 4\sqrt{ \ ^2 + 9}$

$+2(\ + \ - 5)\sqrt{41} + 2(\ - 3)\sqrt{16 + \ ^2} + (\ - 3 \ - 2 \ + 6)\sqrt{ \ ^2 + 2 \ + 26} + 40$

$(\ ) =$ $(\ ) = 3 \ - \ - 3$. $= 3$ $= \ + 1$ 2

$2(\ - 2)$ 4 $(\ - 3)$ 8

4 4 $2(\ + \ - 7)$ 4 $2(\ + \ - 5)$ 4 5

$(2 \ - 5 \ - 7 \ + 17)$ 5 $2(\ - 3)$ 4 $(\ - 3 \ - 2 \ + 6)$

5 Sombor

3 = □ Sombor

$(\ ) = (\ - 1)\sqrt{(\ + \ - 2)^2 + (\ + 2)^2} + (\ - 1)\sqrt{(\ + \ - 2)^2 + (\ + 2)^2}$

$+(\ - 1)(\ - 1)\left(\sqrt{ \ ^2 + 4 \ + 40} + \sqrt{ \ ^2 + 4 \ + 40}\right) + (14 \ - 11 \ - 11 \ + 8)\sqrt{2}$

$(\ ) =$ $(\ ) = 4 \ - 2 \ - 2$. $= 6$ $= \ + \ - 2$

$(\ - 1)$ $(\ + 2)$ $(\ - 1)$ $(\ + 2)$ $(\ - 1)(\ - 1)$

$(\ + 2)$ $(\ - 1)(\ - 1)$ $(\ + 2)$ $2(\ - 1)(\ - 1)$

$(\ - 1)$ $(\ + 2)$ $(\ - 1)$ $(\ + 2)$

Sombor

4 = □ Sombor

$(\ ) = 2\left(\sqrt{(\ + \ - 2)^2 + (\ + 1)^2} + \sqrt{(\ + 1)^2 + (\ + 2)^2}\right) + (\ - 3)\sqrt{(\ + \ - 2)^2 + (\ + 2)^2}$

$+(\ - 1)\sqrt{(\ + \ - 2)^2 + (\ + 2)^2} + (14 \ - 34 \ - 11 \ + 22)\sqrt{2} + 2(\ - 1)$

$\left(\sqrt{ \ ^2 + 2 \ + 26} + \sqrt{ \ ^2 + 4 \ + 29} + \sqrt{61}\right) + (\ - 1)(\ - 3)\left(\sqrt{ \ ^2 + 4 \ + 40} + \sqrt{ \ ^2 + 4 \ + 40}\right)$

$(\ ) =$ $(\ ) = 4 \ - 3 \ - 2$. $= 5$ $= \ + \ - 2$ 2

$(\ + 1)$ $(\ - 3)$ $(\ + 2)$ $(\ - 1)$

$(\ + 2)$ $(2 \ - 2)$ $(2 \ - 2)$ 6 $(2 \ - 2)$

$(\ + 1)$ $(2 \ - 2)$ $(\ + 2)$ $(\ - 1)(\ - 3)$ $(\ + 2)$ 6

$(\ - 1)(\ - 3)$ $(\ + 2)$ 6 $(2 \ - 2 \ - 7 \ + 7)$ 6 2 $(\ + 1)$

$(\ + 2)$ $(\ - 1)$ $(\ + 2)$ $(\ - 4)$ $(\ + 2)$

$(\ ) = \frac{1}{2} (\ ^2 + \ ^2 + \ ^2 - \ ^2 + 2 \ + 2 \ + 2 \ - 10 + \ + \ + \ - 13)\sqrt{2} + 6 \ \sqrt{2 \ ^2 + 6 \ + 5}$

$(\ ) = (\ + \ + \ - 1)$ $(\ ) = \frac{1}{2} (\ + \ + \ - \ + \ + \ + \ + 1).$ $= \ + 1$

$= \ + 2$ $(\ - 1)$ 6

$\frac{1}{2} (\ + \ + \ + \ + \ + \ - 3 \ - 9)$ Sombor

$(\ ) = \frac{1}{2} (\ ^2 + \ ^2 + \ ^2 - \ ^2 + 2 \ + 2 \ + 2 \ - 10 + \ + \ + \ - 13)\sqrt{2} + 6 \ \sqrt{2 \ ^2 + 6 \ + 5}$

8 = , □ Sombor

$(\ ) = \frac{1}{2} (\ ^2 + \ ^2 + 2 \ + 2 \ + \ + \ - 8 \ - 8)\sqrt{2} + 3 \ \sqrt{2 \ ^2 + 6 \ + 5} + \ \sqrt{2 \ ^2 + 2 \ + 1}$

$(\ ) = (\ + \ )$ $(\ ) = \frac{1}{2} (\ + \ + \ + \ )$ $=$ $= \ + 2$

$\frac{1}{2} (\ - 1)$ 3 $(\ + 1)$ $\frac{1}{2} (\ + \ - 2 \ + \ + \ - 6)$

$(\ + 1)$ $(\ + 1)$ $\frac{1}{2} (\ - 1)$ Sombor

$(\ ) = \frac{1}{2} (\ ^2 + \ ^2 + 2 \ + 2 \ + \ + \ - 8 \ - 8)\sqrt{2} + 3 \ \sqrt{2 \ ^2 + 6 \ + 5} + \ \sqrt{2 \ ^2 + 2 \ + 1}$

3 Sombor

Sombor Sombor Sombor

9 -

$(\ \times \ ) = \ ^2 \ (\ )$

$(\ ) = \{ \ _1, \ _2, \ , \ \}$ $(\ ) = \{ \ _1, \ _2, \ , \ \}$ $(\ ) = \{ \ 1 \le \ , \ \le \ \}$ $(\ ) = \{ \ 1 \le \ , \ \le \ \}$

$(\ \times \ ) = \{ \ 1 \le \ \le \ , 1 \le \ \le \ \}$ $(\ \times \ ) = \{ (\ )(\ ) \ 1 \le \ , \ \le \ , 1 \le \ , \ \le \ \}$ $(\ ) =$

$_\times (\ ) = \ (\ ) \ (\ ) = \ (\ ).$ $_1 = (\ )(\ )$ $(\ \times \ )$ $_2 = (\ )(\ )$ $(\ \times \ )$

$(\ \times \ ) = 2 (\ ) (\ )$ $_\times (\ ) = \ _\times (\ )$ $_\times (\ ) = \ _\times (\ )$

$(\ \times \ ) = \sum_{(\ )(\ ) \ \times} \sqrt{ \ _\times{}^2(\ ) + \ _\times{}^2(\ )} = 2 \ \frac{}{2} \sum \sqrt{ \ ^2(\ ) + \ ^2(\ )} = \ ^2 \ (\ )$

4 Sombor

Sombor

Sombor

10 1 $(\ □ \ ) = 2 \ \sqrt{2 \ ^2 + 2 \ + 1} + \left(\frac{1}{2} \ ^3 + \ ^2 + \frac{1}{2} \ - 4 \ ^2 - 2 \ \right)\sqrt{2}$

2 $(\ □ \ ) = \frac{1}{2} \ (\ + 1)(\ + 1)\sqrt{2}$

3 $(\ \times \ ) = (2 \ ^3 - 4 \ ^2 + 2 \ )\sqrt{5} + (2 \ ^3 \ - 4 \ ^2 \ - 6 \ ^3 + 12 \ ^2 + 2 \ - 6 \ )\sqrt{2}$ $(\ > 2)$

4 $(\ \times \ ) = \ (\ - 2)(\ - 1)\sqrt{2}$

5 $(\ ) = 2 \ ^2\sqrt{13 \ ^2 - 10 \ + 2} + \left(\frac{9}{2} \ ^3 \ - 3 \ ^2 \ - 10 \ ^3 + 4 \ ^2 + \frac{1}{2} \ \right)\sqrt{2}$ $(\ > 2)$

6 ( ) $= \frac{1}{2}$ $(3 - 1)^2\sqrt{2}$

7 (×) $= 4\sqrt{17} + (8 + 8 - 48)\sqrt{5} + (8 - 24 - 24 + 80)\sqrt{2}$ (, > 2)

8 (×) $= 8 \sqrt{2}$

9 (×) $= 8 \sqrt{5} + (8 - 12)\sqrt{2}$ (> 2)

10 ( ) $= (32 - 78 - 78 + 200)\sqrt{2} + 8\sqrt{34} + 4\sqrt{53} + (6 + 6 - 32)\sqrt{89}$ (, > 2)

11 ( ) $= 32 \sqrt{2}$

12 ( ) $= (32 - 78)\sqrt{2} + 6 \sqrt{89}$.

1 (□) = (□) $= \frac{1}{2}$ $^2 + \frac{1}{2}$ - □ $_□$ =

$_□$ = + 1 ^{2} - 2

$\left(\frac{1}{2} \quad ^2 + \frac{1}{2} \quad - \quad ^2 - 2\right)$ □ Sombor

2 ^{7} - Sombor ( ) $= \frac{^2}{\sqrt{2}}$ □

(+ 1) - (□) $= \frac{1}{2}$ (+ 1)(+ 1)$\sqrt{2}$

3 × 10 - 1-

(×) = ^{2} ( ) = (- 1)$^2\left[2\sqrt{5} + 2(- 3)\sqrt{2}\right]$

$= (2\ ^3 - 4\ ^2 + 2)\sqrt{5} + (2\ ^3 - 4\ ^2 - 6\ ^3 + 12\ ^2 + 2 - 6)\sqrt{2}$ (> 2)

4 × (2 - 2) - (×) = (- 2)(- 1)$\sqrt{2}$

5 ( ) = ( ) $= \frac{3}{2}$ ^{2} - ^{2} - $\frac{1}{2}$ = 2 - 1

= 3 - 1 ^{2} - 2 ^{2} $\left(\frac{3}{2} \quad ^2 - 4\ ^2 - \frac{1}{2} \quad + \right)$

Sombor

6 (3 - 1) -

( ) $= \frac{1}{2}$ $(3 - 1)^2\sqrt{2}$

7 (×) = (×) = 2(- 1)(- 1) × $_×$ = 1

$_×$ = 4 4 4 2 (4 + 4 - 24) 2

(2 - 6 - 6 + 18) × Sombor

8 × 10 2- (×) = ^{2} ( ) =

$2^2 2 \sqrt{2} = 8 \sqrt{2}$

9 × 10 2- (×) = ^{2} ( ) =

$2^2\left[2\sqrt{5} + 2(- 3)\sqrt{2}\right] = 8 \sqrt{5} + (8 - 12)\sqrt{2}$

10 ( ) = ( ) = 4 - 3 - 3 + 2 = 3 = 8

8 5 4 (2 + 2 - 8) 5

(6 + 6 - 32) 5 (4 - 11 - 11 + 30)

Sombor

11 8- ( ) $= \frac{8^2}{\sqrt{2}} = 32 \sqrt{2}$

12 ( ) = ( ) = 4 - 3 = 5 = 8

2 6 (4 - 11)

Sombor

11 = 1 2 3

( ) > ∑ ( )
= 1

1, 2, 3, , = 1 2 3 (1 ≤ ≤)

( ) ≠ ( ) = (1) + (2) + (3) + + ( ) (1 ≤ ≤) ( )

( ) ( ) ≥ ( ) = ( ) ( ) ( )

( ) ( ) > ( ) $\sqrt{\ ^2(\ ) +\ ^2(\ )} > \sqrt{\ ^2(\ ) +\ ^2(\ )}$ ( ) ( )

( ) > ( ) ( ) = (1) + (2) + (3) + + ( ) ( ) > ∑ ( ).
= 1

1

( ) > (×) + (□)

= (×) (□) ( ) = ( )

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Sombor index is a new topological index based on vertex degree introduced by Gutman in Chemical Graph Theory. In this paper Sombor indices of cartesian products of path with fan graph and wheel graph wheel graph with fan graph and wheel graph fan graph with fan graph and lollipop graph , barbell graph , , and kite graph , with complete graph are discussed Sombor indices of Direct products Cartesian products and Strong products of complete graph path and cycleare are also studied and the exact index values and some relations of sombor indices about product graph are obtained.

Sombor index Cartesian product Direct product Strong product

9

ZHAO Meng-ru ZHOU Ju-ling*

Based on gradually increasing type truncated samples. Firstly obtain the maximum likelihood estimation of the Pareto distribution shape parameter considering the two loss functions and the two prior distributions of shape parameters four Bayes estimation of the distribution shape parameter is concluded. It is found from the numerical simulation results that the mean square error of the four Bayes estimates is less than the maxi mum likelihood estimate. Among them when the loss function is a quadratic loss function and the prior distribution of the shape parameter is a conjugate prior distrik W fun prior dist and the estimation effect is more ideal and the example analysis is consistent with the numerical simulation re sults. Secondly under the quadratic loss function tion of the shape parameters of the Pareto distribution are given.

Gradually increasing of type truncation Pareto distribution Quadratic loss Q-symmetri entropy loss Bayes estimates