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摘　要:

1 预备知识

定义 1[4] X X o l

$$F(x8\theta a) \quad (ax \quad)^{\theta} x;$$

$$f(x8a\theta) \quad \theta a^{\theta} x^{(\theta(\quad)} x;$$

a θ a； θ；

n o l m 9 n

R_j j m X n

R X

n R R m

X_m R_m n m $\sum_{j}^{m}$ R_j $R_j = 0$

引理 1[10] o l

证明 X (X X X_n) o l

Γ(α β)

$$\pi(\theta) \quad \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha} \quad e^{\beta\theta}$$

$$f(x|\theta) \quad \prod_{j}^{n} f(x_j) \quad \theta^{n} a^{\theta n} \prod_{j}^{n} x_j^{(\theta(\quad)}$$

θ

$$\pi(\theta|x) \quad \frac{\pi(\theta)f(x|\theta)}{\int^{(\infty} \pi(\theta)f(x|\theta)d\theta} \quad \frac{\left(\beta \quad n\,\mathrm{ik}\,a\,(\sum_{j}^{n}\mathrm{ik}\,x_j\right)^{n(\alpha}}{\Gamma(n(\alpha)}\theta^{n(\alpha} \quad e^{\theta\left(\beta \quad n\,\mathrm{ik}\,a\,(\sum_{j}^{n}\mathrm{ik}\,x_j\right)}$$

$$\theta\Big|x \quad \Gamma\left(n(\alpha, \beta \quad n\,\mathrm{ik}\,a\,(\sum_{j}^{n}\mathrm{ik}\,x_j\right)$$

Γ(α, β)

$$\pi(\theta) \quad \overline{\theta}$$

$$\pi(\theta) \quad \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha} \quad e^{\beta\theta}$$

2 形状参数的极大似然估计和Bayes估计

2.1 形状参数的极大似然估计

o l

$$\overset{*}{L}(a\theta|x) \quad M\prod_{j}^{m} f(x_j)[$$

$$M \quad n(n \quad R\,)(n \quad R \quad R\,) \quad \left(n \quad \sum_{j}^{m} \left(R_j\,(\quad)\right)\right)$$

$$\mathrm{ik}\,L(a\,\theta|x) \quad \mathrm{ik}\,M\,(\quad m\,\mathrm{ik}\,\theta \quad m\,\mathrm{ik}\,a \quad \sum_{j}^{m}\left[\theta(\quad(\quad R_j)(\quad\right]\mathrm{ik}\left(\frac{x_j}{a}\right)$$

$$\theta_M \quad \frac{m}{T}$$

$$T \quad \sum_{j}^{m}(\quad(\quad R_j)\,\mathrm{ik}\left(\frac{x_j}{a}\right)$$

2.2 二次损失函数下形状参数的Bayes估计

定义 2[15]

$$S\,(\theta\,\delta) \quad \frac{(\delta \quad \theta)}{\theta} \qquad 0$$

引理 2[15] o l θ p

$$\delta \quad \frac{E(\theta \quad |X)}{E(\theta \quad |X)}$$

δ θ

定理 1 0 $X \quad (X \;\; X \quad X_n)$ o l

π (θ) p

$$\delta \quad \frac{m}{T}$$

$$E(\theta\ |X)\quad {}^{(\infty}\theta\ \ h(a\theta|x)d\theta\quad {}^{(\infty}\theta\ \frac{(\beta(\ T)^{\alpha(\ m}}{\Gamma(\alpha(\ m)}\theta^{\alpha(\ m}\quad e^{\theta(\beta(\ T)}d\theta$$

$$\frac{(\beta(\ T)^{\alpha(\ m}}{\Gamma(\alpha(\ m)}\ {}^{(\infty}\theta^{\alpha(\ m}\quad e^{\theta(\beta(\ T)}d\theta\quad \frac{\beta(\ T}{\alpha(\ m}$$

$$E(\theta\ |X)\quad {}^{(\infty}\theta\ \ h(a\theta|x)d\theta\quad {}^{(\infty}\theta\ \frac{(\beta(\ T)^{\alpha(\ m}}{\Gamma(\alpha(\ m)}\theta^{\alpha(\ m}\quad e^{\theta(\beta(\ T)}d\theta$$

$$\frac{(\beta(\ T)^{\alpha(\ m}}{\Gamma(\alpha(\ m)}\ {}^{(\infty}\theta^{\alpha(\ m\quad 0}e^{\theta(\beta(\ T)}d\theta\quad \frac{(\beta(\ T)}{(\alpha(\ m\quad)(\alpha(\ m\quad)}$$

p

$$\delta\quad \frac{E(\theta\ |X)}{E(\theta\ |X)}\quad \frac{\alpha(\ m}{\beta(\ T}$$

2.3 Q对称熵损失函数下形状参数的Bayes估计

定义3[16]

$$S(\theta\delta)\quad \left(\frac{\theta}{\delta}\right)^{q}(\ \left(\frac{\delta}{\theta}\right)^{q}$$

引理3[16] o l p

$$\delta\quad \left(\frac{E(\theta^{q}|X)}{E(\theta\ ^{q}|X)}\right)^{\overline{\ \ }q}$$

δ θ q；

定理3 $X\quad (X\ \ X\qquad X_n)$ o l

$\pi(\theta)$ p

$$\delta\quad \left[\frac{\Gamma(m(\ \ q)}{\Gamma(m\quad q)}\right]^{\overline{\ \ }q}\overline{T}$$

证明 $h(a\theta|x)\quad \frac{T^{m}}{\Gamma(m)}\theta^{m}\quad e^{\ \theta T}$

$$E(\theta^{q}|X)\quad {}^{(\infty}\theta^{q}h(a\theta|x)d\theta\quad {}^{(\infty}\theta^{q}\frac{T^{m}}{\Gamma(m)}\theta^{m}\quad e^{\ \theta T}d\theta$$

$$\frac{T^{m}}{\Gamma(m)}\ {}^{(\infty}\theta^{m(\ q}\quad e^{\ \theta T}d\theta\quad \frac{\Gamma(m(\ \ q)}{\Gamma(m)T^{q}}$$

$$E(\theta\ ^{q}|X)\quad {}^{(\infty}\theta\ ^{q}h(a\theta|x)d\theta\quad {}^{(\infty}\theta\ ^{q}\frac{T^{m}}{\Gamma(m)}\theta^{m}\quad e^{\ \theta T}d\theta$$

$$\frac{T^{m}}{\Gamma(m)}\ {}^{(\infty}\theta^{m\quad q}\quad e^{\ \theta T}d\theta\quad \frac{\Gamma(m\quad q)}{\Gamma(m)T\ ^{q}}$$

p

$$\delta\quad \left[\frac{E(\theta^{q}|X)}{E(\theta\ ^{q}|X)}\right]^{\overline{\ \ }q}\quad \left[\frac{\Gamma(m(\ \ q)}{\Gamma(m\quad q)}\right]^{\overline{\ \ }q}\overline{T}$$

定理4 $X\quad (X\ \ X\qquad X_n)$ o l

$\pi(\theta)$ p

$$\delta\quad \left[\frac{\Gamma(\alpha(\ \ m(\ \ q)}{\Gamma(\alpha(\ \ m\quad q)}\right]^{\overline{\ \ }q}\overline{\beta(\ \ T}$$

证明 $h(a\theta|x) \quad \frac{(\beta(T)^{\alpha(m}}{\Gamma(\alpha(m)}\theta^{\alpha(m} \quad e^{\theta(\beta(T)}$

$$E(\theta^q|X) \quad \int^{(\infty}\theta^q h(a\theta|x)d\theta \quad \int^{(\infty}\theta^q\frac{(\beta(T)^{\alpha(m}}{\Gamma(\alpha(m)}\theta^{\alpha(m} \quad e^{\theta(\beta(T)}d\theta$$

$$\frac{(\beta(T)^{\alpha(m}}{\Gamma(\alpha(m)}\int^{(\infty}\theta^{\alpha(m(q} \quad e^{\theta(\beta(T)}d\theta \quad \frac{\Gamma(\alpha(m(q)}{\Gamma(\alpha(m)(\beta(T)^q}$$

$$E(\theta^{q}|X) \quad \int^{(\infty}\theta^{q}h(a\theta|x)d\theta \quad \int^{(\infty}\theta^{q}\frac{(\beta(T)^{\alpha(m}}{\Gamma(\alpha(m)}\theta^{\alpha(m} \quad e^{\theta(\beta(T)}d\theta$$

$$\frac{(\beta(T)^{\alpha(m}}{\Gamma(\alpha(m)}\int^{(\infty}\theta^{\alpha(m \quad q} \quad e^{\theta(\beta(T)}d\theta \quad \frac{\Gamma(\alpha(m \quad q)}{\Gamma(\alpha(m)(\beta(T)^{q}}$$

p

$$\delta \quad \left[\frac{E(\theta^q|X)}{E(\theta^{q}|X)}\right]^{\overline{q}} \quad \left[\frac{\Gamma(\alpha(m(q)}{\Gamma(\alpha(m \quad q)}\right]^{\overline{q}}\frac{}{\beta(T}$$

3 形状参数的多层Bayes估计和E-Bayes估计

o l

p p

3.1 形状参数的多层Bayes估计

$\Gamma(\alpha\ \beta)$ $\alpha\ \beta$

$U(\quad)\ U(\quad c)\quad \alpha\ \beta$

$\pi(\alpha) \quad \pi(\beta)\ \frac{}{c}$

9 β 9 c(c;) c

定理5 0 X (X X X_n)

o l $\pi(\theta)$ $\alpha\ \beta$

p

$$\delta_{HB} \quad \frac{\int^c\frac{\beta^\alpha\Gamma(\alpha(m \quad)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}{\int^c\frac{\beta^\alpha\Gamma(\alpha(m \quad)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}$$

证明 $\alpha\ \beta$

$$g(\theta) \quad \int^c\pi(\theta)\pi(\alpha)\pi(\beta)d\alpha d\beta \quad \frac{}{c} \quad \int^c\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^\alpha \quad e^{\beta\theta}d\alpha d\beta$$

$$h_0(a\theta|x) \quad \frac{g(\theta)L(a\theta|x)}{\int_\infty^{(\infty}g(\theta)L(a\theta|x)d\theta} \quad \frac{\int^c\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha(m} \quad e^{(\beta(T)\theta}d\alpha d\beta}{\int^c\frac{\beta^\alpha\Gamma(\alpha(m)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}$$

$$E(\theta \quad |X) \quad \int_\infty^{(\infty}\theta \quad h_0(a\theta|x)d\theta \quad \frac{\int^c\int^{(\infty}\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha(m} \quad e^{(\beta(T)\theta}d\theta d\alpha d\beta}{\int^c\frac{\beta^\alpha\Gamma(\alpha(m)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta} \quad \frac{\int^c\frac{\beta^\alpha\Gamma(\alpha(m \quad)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}{\int^c\frac{\beta^\alpha\Gamma(\alpha(m)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}$$

$$E(\theta|X) \quad \int_{\infty}^{(\infty}\theta \quad h_0(a\,\theta|x)d\theta \quad \frac{\int^{c}\int^{(\infty}\frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha(m} \quad e^{(\beta(T)\theta}d\theta d\alpha d\beta}{\int^{c}\frac{\beta^{\alpha}\Gamma(\alpha(m)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}$$

$$\frac{\int^{c}\frac{\beta^{\alpha}\Gamma(\alpha(m \quad)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}{\int^{c}\frac{\beta^{\alpha}\Gamma(\alpha(m)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}$$

p

$$\delta_{HB} \quad \frac{E(\theta|X)}{E(\theta|X)} \quad \frac{\int^{c}\frac{\beta^{\alpha}\Gamma(\alpha(m \quad)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}{\int^{c}\frac{\beta^{\alpha}\Gamma(\alpha(m \quad)}{\Gamma(\alpha)(\beta(T)^{\alpha(m}}d\alpha d\beta}$$

3.2 形状参数的E-Bayes估计

p p

定义4[17] $\theta_B(a\,b)$ θ_{EB} $\int_D \theta_B(a\,b) \quad p(a\,b)dadb$ p

$\int_D \theta_B(a\,b) \quad p(a\,b)dadb \; 9 \; \infty$ D a b $p(a\,b)$ a b $\theta_B(a\,b)$

p

定理6 0 X $(X \; X \quad X_n)$ o l

$\pi(\theta)$ $\alpha \; \beta$

p

$$\delta_{EB} \quad \frac{0(\quad m}{c} ik\Big(\quad (\frac{c}{T}\Big)$$

证明 $\alpha \; \beta$

$$\pi(\alpha \; \beta) \quad \pi(\alpha)\pi(\beta) \quad \frac{}{c}$$

p

$$\delta_{EB} \quad \int_D \delta \quad \pi(\alpha \; \beta)d\alpha d\beta \quad \int^{c}\frac{\alpha(m}{\beta(T}\frac{}{c}d\alpha d\beta$$

$$- \quad (\alpha(m \quad) \int^{c}\frac{}{\beta(T}d\beta d\alpha \quad \frac{}{c}ik\Big(\quad (\quad \Big)$$

α β q

p o l

表 1 n,m 取不同值时，$\theta_M,\delta_{11},\delta_{12}$ 的模拟结果

n	m	θ_M	δ	δ
		0	00	
		0	0	
		0		0
		0		
		0		0
	0			0 0
	0		0	
	0			
		0	0	
		0	0	

表 2 n,m 取不同值时，δ_{21},δ_{22} 的模拟结果

n	m	δ	δ
		0	0
		0	0 0
		0	
			0
			0 0
			0
	0		0
	0		0
	0		0
			0
			0

α β c

o l p p

0

表3 n,m取不同值时,δ_{HB},δ_{EB}的模拟结果

n	m	δ_{HB}		δ_{EB}	
		0			
		0			
		0			
	0				
		0	0		0

p

p δ δ

δ δ p

0 n m p

p p δ

δ

o l p

5 实例分析

a θ o l